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CHARACTERIZATION OF ELASTIC SHELLS BY THE USE OF THE WAVELET TRANSFORM.

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Abstract

This paper concerns the characterization of elastic targets immersed in a fluid and submitted to an acoustic impulse. Time-frequency methods have already been used in the case of scatterers of simple geometric shape. We have chosen the wavelet transform for its particular properties, such as linearity and local analysis at $\Delta f/f=c^{ste}$. We have developped an algorithm based on the behavior of the phase of the transform, which enables us to extract modulation laws (related to the dispersion law of the phase velocity), even for close echoes. In the case of spherical elastic shells, we have applied this method on both experimental and computer-generated signals and have pointed out the good relation between theoretical and experimental results.

1- Introduction

We are interested in the characterization of targets of simple geometric shape from the analysis of their acoustic response. This problem requires on the one hand the knowledge of the mechanisms of the surface waves generation and on the other hand, the choice of relevant parameters for the identification [1-11].

In this paper, we will limit ourselves to the study of signals back-scattered by spherical elastic shells in water submitted to an acoustic impulse. The parameters usually estimated are: kind of waves, arrival time of waves packets, group velocity, resonance frequencies and maxima of energy.

From a general point of view, the back-scattered pressure field can be decomposed into a geometric pressure (geometric wave) and surface pressure (surface wave and its successive echos around the target).

If the surface or packets waves are the same for targets of the same shape, the dispersion law of their phase velocity changes with respect to their thickness and their chemical composition. Targets characterization (ie: detection and identification of surface waves), requires time-frequency methods. We will focus on the estimation of frequency modulation laws of the waves packets by the use of the wavelet transform. The use of this method is justified by its linearity and localization properties.

2- Description of the physical problem.

This problem has been mainly approached by the works of Überall, Flax, Derem [1,12,13].

In particular, Überall has proved the relation between the winding of the surface waves around the scatterer and the resonance phenomenon [12]. For instance, we have a resonance for n+1/2 wave length for a spherical shell, and n wave length for a cylinder one.

The surface waves can be separated in two classes: the waves having a support in the fluid media (Stoneley, Franz waves called creeping waves), and the ones where the support is on the elastic scatterer (Rayleigh and whispering waves). Franz and whispering waves are related to the geometry of the target.

For an incident plane wave, the pressure scattered by a spherical elastic shell is given by:

$$P_d(r,t)=P_0\int e^{j\omega t}\int i^n(2n+1)\frac{N_n(kr)}{D_n(kr)}\,h_n^{(1)}(kr)P_n(\cos\theta)\,dk\,d\omega, \quad (1)$$

P_0 represents the amplitude of the incident wave;

N_n and D_n are functions depending of the target geometry. They are written as (6x6) determinant taking into account the boundary conditions [3,8]. The resonances coming from the interfaces (shell) corresponds to the singularities of these functions [3,12];

$h_n^{(1)}$ represents the spherical Hankel functions of the first kind;

P_n are the Legendre functions.

The extraction of the resonance position of the target is done classically, in the case of harmonic source, by spectral analysis, from the modulus of the farfield form function [3]:

$$|F_\infty(kr,\theta)| = \frac{2}{kr}\left|\sum_{n=0}^{\infty}(-i)^n(2n+1)\,A_n(kr)\,P_n(\cos\theta)\right| = \frac{1}{kr}\left|\frac{P_{diff}(r)}{P_{inc}(r)}\right|$$

The integral of the expression (1) on a path Γ can be decomposed into a sum of integrals due to the different branch-points [14]. Each integration around these points corresponds to a surface wave. These waves have their own time and frequency behavior.

3- Continuous time-scale transform: Continuous wavelet transform.

3-1: recalls and definitions[15,16]

The continuous wavelet transform of an arbitrary signal s is obtained by the scalar product between this signal and elementary functions, called wavelets. These functions are translated [b] and dilated copies [a] of a basic function g (named analyzing wavelet) in the time and scale half-plane (b,a). The wavelet transform is given by :

$$S(b,a) = \langle g_{b,a} \mid s \rangle = \int \bar{g}_{b,a}\, s(t)\, dt = a^{\alpha} \int \bar{g}\left(\frac{t-b}{a}\right) s(t) dt$$

$$b \in R, \quad a > 0$$

$\bar{g}$ denotes the complex conjugate of g; α depends of the choice of the normalization (α=1/2 with the normalization L_2).

The transform is inversible if we can define a constant c_g depending only on the wavelet:

$c_g = 2\pi \int \frac{|\hat{g}(\omega)|^2}{\omega} d\omega < \infty$, $\hat{g}$ represents the Fourier transform of g.

This necessary condition on the wavelet is called the admissibility one. In practice, it means that g is of finite energy, g is of zero mean.

The inversion formula is:

$$s(t) = \frac{1}{c_g} \int S(b,a)\, g_{b,a}(t) \frac{db\, da}{a^{2+\alpha}},$$

$\frac{db\, da}{a^2}$ is the invariant measure under translation and dilation in the half-plane.

The frequency localization of $g_{b,a}(t)$ depends linearly of a, so the decomposition induced will be performed at $\frac{\Delta\omega}{\omega}$ = Constant.

3-2: Extraction of amplitude and frequency modulation laws [17-19].

We present here, a method essentially based on the stationary phase approximation. The particularity of this method lives in the asymptotic conditions of the signal with respect to the analyzing wavelet, coming from the local properties of the wavelet.

One may define the asymptotism in the following way:

Let $s(t)=A_s(t)\cos(\Phi_s(t))$ an asymptotic signal, then the analytic associated signal is: $s_a(t)=A_s(t)\exp(i\Phi_s(t))$.

In this part, we will focus on this class of signals. Let:

$$s(t)=A_s(t)\exp(i\Phi_s(t)) \quad ; \quad g(t)= A_g(t)\exp(i\omega_o t)$$

Then:

(2) $$S(b,a)=\frac{1}{a^{\alpha}} \int A_s(t) A_g\left(\frac{t-b}{a}\right) \exp\left(i\left(\Phi_s(t)-\omega_o\left(\frac{t-b}{a}\right)\right)\right) dt$$

The stationary phase criteria at a point $\tau(a)$ leading to an approximation of the integral (2) is:

$$\frac{d\Phi_s(t)}{dt} = \frac{\omega_o}{a} \quad \text{en } t=\tau(a)$$

If we suppose the signal to be asymptotic with respect to the wavelet at $\tau(a)$, ie: $A_s(t)$ varies slowly in front of $A_g\left(\frac{t-b}{a}\right)$, we have:

(3)

$$S(b,a) = \frac{1}{a^{\alpha}} A_s(\tau(a)) \int A_g\left(\frac{t-b}{a}\right) \exp\left[i\left(\Phi_s(t)-\omega_o\left(\frac{t-b}{a}\right)\right)\right] dt + \varepsilon_1$$

With a second order limited expansion of the phase of the integral (3) at the same point $\tau(a)$, we obtaine:

$$S(b,a) = \frac{1}{a^{\alpha}} A_s(\tau(a)) \exp\left[i\left(\Phi_s(\tau(a))-\omega_o\left(\frac{\tau(a)-b}{a}\right)\right)\right] \times$$
$$\times \int A_g\left(\frac{t-b}{a}\right) \exp\left[i\left(\frac{1}{2}(t-\tau(a))^2 \frac{d^2\Phi_s(\tau(a))}{dt^2}\right)\right] dt + \varepsilon_1 + \varepsilon_2$$

Suppose $A_g(t)=\exp(-\frac{1}{2}t^2)$ (case of the Morlet wavelet), then S(b,a) is a gaussian integral. We note:

$$\Phi''_s = \frac{d^2\Phi_s(\tau(a))}{dt^2} \quad , \quad S(b,a) = M(b,a)\exp(i\Phi_T(b,a))$$

Then:

$$\Phi_T(b,a) = \Phi_s(\tau(a))-\omega_o\left(\frac{\tau(a)-b}{a}\right) + \frac{1}{2}\mathrm{Arctan}(a^2\Phi''_s) +$$
$$+ \frac{1}{2}(b-\tau(a))^2 \frac{\Phi''_s}{1+a^4\Phi''^2_s}$$

$$M(b,a)=\frac{\sqrt{2\pi}\, A_s(\tau(a))}{a^{\alpha}(1+a^4\Phi''^2_s)^{1/4}} \exp\left(\frac{1}{2}\frac{a^4\Phi''^2_s}{(1+a^4\Phi''^2_s)}\left(\frac{b-\tau(a)}{a}\right)^2\right)$$

and $\frac{\partial\Phi_T(b,a)}{\partial b} = \frac{\omega_o}{a}$ at $b=\tau(a)$

M(b,a) is maximum

$S(\tau(a),a)=C^{st}\, s(t)$

a-Interpretation

We will call 'ridge' the curve $b=\tau(a)$ and the restriction of the wavelet transform along the ridge will be call 'skeleton'.

The ridge detection consists on the search of the points over the half-plane (b,a) such as: $\frac{\partial \Phi_T(b,a)}{\partial b} = \frac{\omega_0}{a}$. This set of points defines a curve where the modulus is maximum.

The ridge and the skeleton have the following properties:

-$S(\tau(a),a)$ is equal to s(t) up to one known coefficient.

-The ridge gives an estimation of the frequency modulation law.

-The modulus of the skeleton gives an estimation of the amplitude modulation law.

b-Error estimation.

The error majoration is:

$$\varepsilon_1 < K_1 \, . \, a \, . \|A'_s(\tau(a))\|_\infty$$
$$\varepsilon_2 < K_2 \, . a^3 . |A_s(\tau(a))| \, . \, \|\Phi_s^{(3)}(\tau(a))\|_\infty$$

Remark: One of the conditions required for this approximation is the asymptotism of the signal with respect to the wavelet. In practice, this condition is controlled since we can choose the wavelet. A few asymptotic signal will require a wavelet well localised in time. This method compared to the one used in [11] can be applied to wave packets containing few oscillations. This method can be applied to impulse signals scattered by elastic targets of simple geometric shape.

4- Time-scale analysis of echoes scattered by a spherical elastic shell.

We consider spherical scatterers immersed in water. The thickness of the shells is small with ratio of radii r_i/r_e of 0.9 (Fig. 1,3) , 0.67 (Fig.2) and 0.96 (Fig. 4), (r_i, r_e : internal, external radius r_e=3cm). They are made in duralumin (ie: alloying of 95% aluminium and 3,5% copper), except for the last one which is made in nickel-molybdene. The transducers used for experiments have either a central frequency of 500kHz with a band-width at -3dB of 400kHz (Fig. 1,2) or a central frequency of 250kHz (Fig 3,4) with a band-width of 150kHz.

We have applied a wavelet analyzis both on experimental and simulated signals, but we present here experimental results. For each analyzis we dispose of two complementary informations on the wavelet transform: the phase (b) and the modulus (c). The impulse response of shells is displayed in (a).

The modulus of the transform is shown in linear scale with a density of points. The modulus is maximum (minimum) when it is black (white). The phase representation is between $(0, 2\pi)$. The ridge is represented by black dots over the phase diagram. The abscissa represents the translation parameter (time). The ordinate represents the dilation parameter in hyperbolic scale (linear in frequency), decomposed in 150 voices. The analyzing wavelet (standard Morlet wavelet) has been defined with the following caracteristics: ω_0=5.336, and g(t)=0, for a=1, when $|g(t)| \leq 10^{-5}$.

The sampling rate of the time signal is respectively 10^7Hz, 1.810^6Hz, 10^7Hz, 10^7Hz. The analysis is performed with a frequency range of [833.33kHZ to 208.33kHz], [900kHz to 56,25] [625kHz to 156kHz] and [500kHz to 78kHz]

Roughly speaking, the dispersion of the shell increases the inclination and the attenuation of waves packets during time. The dispersion law of the phase velocity can be obtained either from the modulus or from the phase of the wavelet transform:

- by extraction of the maxima of energy of the waves packets (whispering echoes) for each scale (analysis of the modulus);
- by extraction of the ridge (fig. 1-4.b) (analysis of the phase).

The analysis of two successive waves packets of the same type allows us to define the dispersion law of the group velocity. The problem is sometimes the measure error generated by the ambient noise during the experiment. In this case, even the wavelet transform has shown a good efficacy [17-19], it is necessary to combine filtering methods with the wavelet transform.

We can see on the figures c in accordance with the theoretical study of the scattering, the specular echo or geometric contribution (Fig: 2, 4) (1^{st} waves packet) and the surface contribution (Fig: 1- 4). This contribution for the shell of ratio of radii 0.9 (fig.1.c) is the whispering wave after one travel around the target (1^{st} waves packet) and after a 2^{nd} travel (3^{th} waves packet). The 2^{nd} waves packet is not dispersive. For each echo, the ridge gives an estimation of the dispersion law of the phase velocity (fig 1.b). The horizontal ridges are characteristics of the non-dispersive echo.

The figure 2 represents the wavelet transform of a signal scattered by a duralumin shell of r_i/r_e=0.67. The echos structure is similar to the figure 1. The modulation law is representative of the thickness of the shell. The frequency of the ridge is over 366kHz to 669kHz.

The figure 3 displays the analysis of the response of the same shell than figure 1, but with a different transducer. The modulation law varies with respect to the frequency of the excitation. The frequency of the ridge is here, over 275kHz to 500kHz.

The figure 4 represents the wavelet transform of a signal backscattered by a nickel-molybdene shell of r_i/r_e=0.96. Again, the echos structure is similar to fig 1. The analysis of this figure shows the relation between the modulation law and the shell chemical composition. Ridge frequency is 223kHz to 398kHz.

5- Conclusion

The dispersive behavior of the surface waves has led us to choose a time and scale method. By an another way the separation of close echoes requires a linear method.

The ridge associated to the continuous wavelet transform seems to be a promising tool for the systematic study of this type of waves. We may considered the response of the target as an acoustic signature with respect to the analyzing wavelet.

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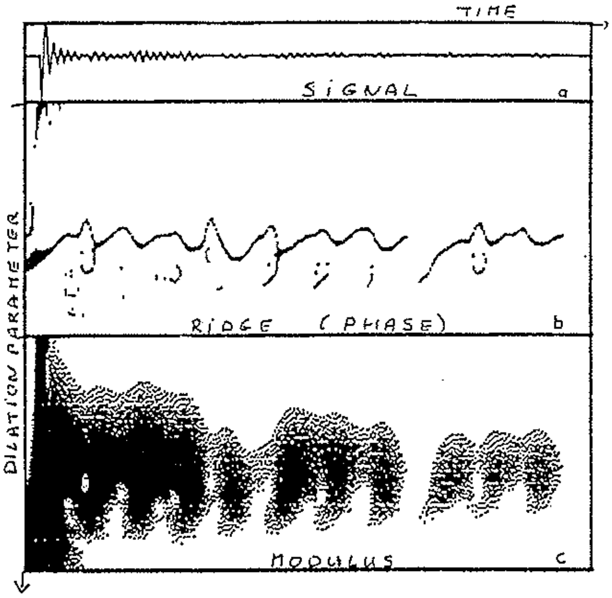

Fig. 2: Impulse response of a thin spherical shell of duralumin with an external radius r_e= 0.03m. r_i/r_e= 0.67 (r_i is the internal radius).Central frequency=500kHz Ridge frequency is 275kHz to 500kHz

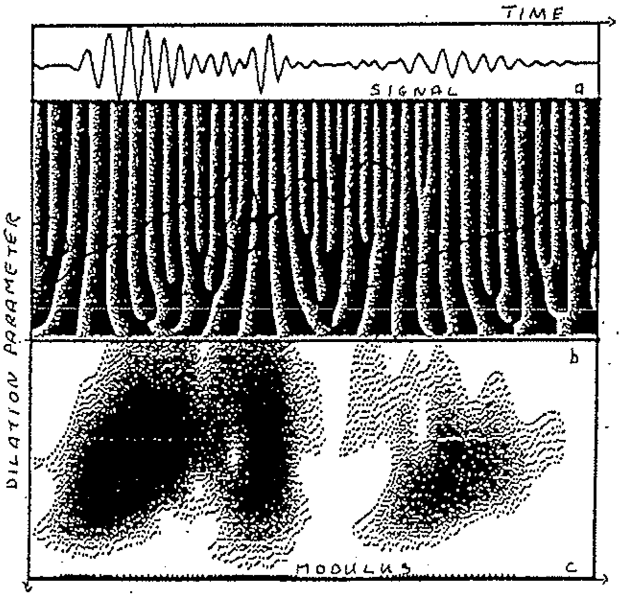

Fig. 1: Impulse response of a thin spherical shell of duralumin with an external radius r_e= 0.03m. r_i/r_e= 0.9 (r_i is the internal radius). (whispering waves: 1 and 2). Central frequency=500kHz Ridge frequency is 366kHz to 669kHz

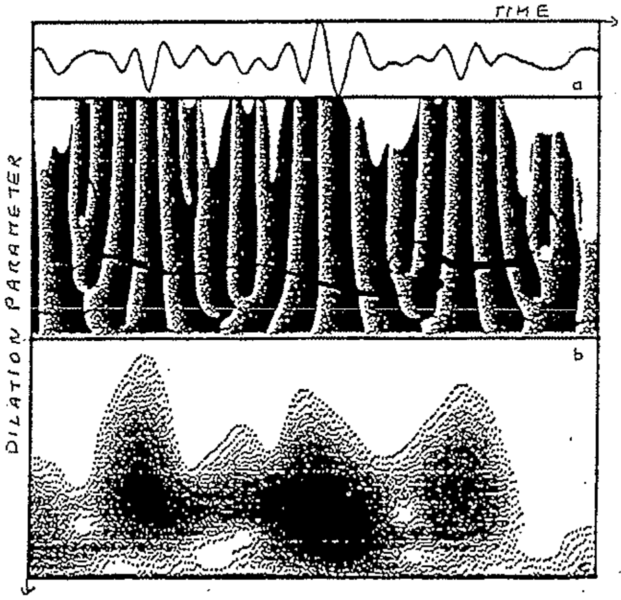

Fig. 3: Impulse response of a thin spherical shell of duralumin with an external radius r_e= 0.03m. r_i/r_e= 0.9 (r_i is the internal radius). Central frequency=250kHz Ridge frequency is 223kHz to 398kHz

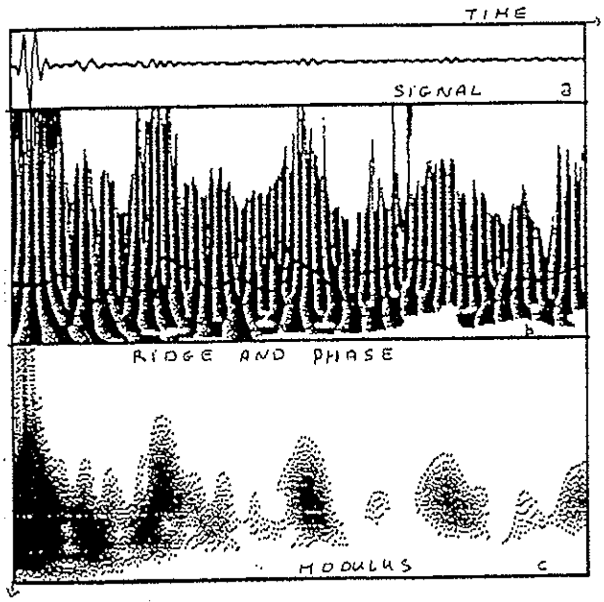

Fig. 4: Impulse response of a thin spherical shell of nickel-molybdene with an external radius r_e= 0.03m. r_i/r_e=0.96 (r_i is the internal radius).Central frequency=250kHz Ridge frequency is 136kHz to 401kHz

References

[1] L. Flax, L. R. Dragonette, H. Überall, 1978, "Theory of elastic resonance excitation by sound scattering", J. Acoust. Soc. Am., 63 (3), 723-731.

[2] A. Derem, 1980, "Diffusion acoustique par des corps cylindriques immergés: aspects généraux, Rev. Cethedec 72, 47-60.

[3] N. Gespa, "La diffusion acoustique par des cibles élastiques de forme géométrique simple. Théorie et expériences", CEDOCAR Ed, (1987), Paris

[4] N.C. Yen, 1987, Time and frequency representation of acoustic signals by means of the Wigner distribution function: implementation and interpretation, J;Acoust; Soc. Am. 81 (6), 1841-1850.

[5] D. Brill, G. Gaunaurd, 1987, "Resonance theory of elastic waves ultrasonically scattered from an elastic sphere", J. Acoust. Soc. Am., 81, 1-21.

[6] M. Talmant, G. Quentin, J.L. Rousselot, Subrahmanyam J.V., H. Überhall, 1988, "Acoustic resonances of thin cylindrical shells and the resonance scattering theory", J. Acoust. Soc. Am., 84(2), 681-688.

[7] J. L. Rousselot, A. Gérard, J.P. Sessarego, J. Sageloli, 1988, "Réponse basse fréquence d'une coque sphérique mince immergée", Acustica, 66, 203-213.

[8] "Electromagnetic and Acoustic Scattering. Detection and Inverse Problems: 31 mai-3 Juin 1988", World Scientific, (1989), Singapore.

[9]Quentin G., Derem A. and poiree B., 1989, Group velocity and acoustical resonances J. Phys. 50, 1943-1952

[10] J.P. Sessarego, J. Sageloli, P. Flandrin, M. Zakharia 1989, "Time-frequency analysis of signal related to scattering problems in acoustics. Part I:Wigner-Ville analysis of echoes scattered by a spherical shell", (Wavelet, time-frequency methods and phase space, Springer-Verlag, Berlin, 147-153.

[11]: G. Saracco, C Gazanhes, J. Sageloli, J.P. Sessarego, J.P., 1989, "Analyse temps-échelle de la diffusion acoustique par des coques sphériques en régime impulsionnel", Journ. Acoust., 1990, vol 3(4)p.381- 391

[12] H. Überall, L.R. Dragonette, L. Flax, 1977, "Relation between creeping waves and normal modes of vibration of a curved body", J. Acoust. Soc. Am., 61 (3), 711-715.

[13] A. Derem, 1987, "Théorie de la matrice S et transformation de Sommerfeld-Watson", (Ch.9, N. Gespa, La diffusion acoustique par des cibles élastiques de forme géométrique simple), CEDOCAR edit, Paris, 189-280.

[14] Saracco G., 1989, "Propagation acoustique en régime harmonique et transitoire à travers un milieu inhomogène: Méthode asymptotique et transformation en ondelettes", Thèse, U.E.R. II, Marseille-Luminy.

[15] A. Grossmann, J. Morlet, 1984, "decomposition of Hardy functions into square integrable wavelets of constant shape", Soc. Int. Am. Math., Journ. of Math. Analys., vol n° 15, 723-736.

[16] Proceed "Wavelet, Time-frequency methods and Phase Space", Marseille-Luminy, déc 1987, J.M. Combes, A. Grossmann, P. Tchamitchian Eds, Springer-Verlag, (1989), Berlin.

[17]: B. Escudié, A. Grossmann, R. Kronland-Martinet, B, Torresani, 1989, "Analyse par ondelettes de signaux asymptotiques, Emploi de la phase stationnaire", 12ème Colloq. G.R.E.T.S.I. (Juan-les-pins)

[18] P. Guillemain, "Analyse et modelisation de signaux acoustiques. Application de la transformée en ondelettes." Report of D.E.A. Marseille 1990

[19] P. Guillemain, R. Kronland-Martinet, B. Martens, "Estimation of spectral lines with the help of the wavelet transform. Application in N.M.R. spectroscopy", to appear in proceed. "wavelet and application", Marseille 1990.